Complete relativistic second-order dissipative hydrodynamics from the entropy principle

Amaresh Jaiswal, Rajeev S. Bhalerao and Subrata Pal
Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India
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We present a new derivation of relativistic dissipative hydrodynamic equations, which invokes the second law of thermodynamics for the entropy four-current expressed in terms of the single-particle phase-space distribution function obtained from Grad's 14-moment approximation. This derivation is complete in the sense that all the second-order transport coefficients are uniquely determined within a single theoretical framework. In particular, this removes the long-standing ambiguity in the relaxation time for bulk viscosity thereby eliminating one of the uncertainties in the extraction of the shear viscosity to entropy density ratio from confrontation with the anisotropic flow data in relativistic heavy-ion collisions. We find that in the one-dimensional scaling expansion, these transport coefficients prevent the occurrence of cavitation even for rather large values of the bulk viscosity estimated in lattice QCD.

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Relativistic fluid dynamics has been quite successful in explaining the various collective phenomena observed in cosmology, astrophysics and the physics of high-energy heavy-ion collisions. The earliest theories of relativistic dissipative hydrodynamics by Eckart [1] and Landau-Lifshitz [2] were based on the assumption that the entropy four-current is first order in dissipative quantities, which led to parabolic differential equations that suffered from acausality. The second-order Israel-Stewart (IS) theory [3] with the entropy current quadratic in dissipative quantities led to hyperbolic equations and thus restored causality.

Application of the second-order viscous hydrodynamics to high-energy heavy-ion collisions has evoked widespread interest ever since a surprisingly small value for the shear viscosity to entropy density ratio η/s was estimated from the analysis of the elliptic flow data [4]. Indeed the estimated η/s was close to the conjectured lower bound $\eta/s|_{\rm KSS} = 1/4\pi$ [5, 6]. This led to the claim that the quark-gluon plasma (QGP) formed at the Relativistic Heavy-Ion Collider (RHIC) was the most perfect fluid ever observed. A precise estimate of η/s is vital to the understanding of the properties of the QCD matter.

In this Communication, we provide a solution to one of the major uncertainties that hinders an accurate extraction of the viscous corrections to the ideal fluid behavior, namely the inadequate knowledge of the second-order transport coefficients. In the standard derivation of second-order evolution equations for dissipative quantities from the requirement of positive divergence of the entropy four-current, the most general algebraic form of the entropy current is parameterized in terms of unknown thermodynamic coefficients [3]. These coefficients which are related to relaxation times and coupling lengths of the shear and bulk pressures and heat current, however, remain undetermined within the framework of thermodynamics alone [7]. While kinetic theory for massless particles [8] and strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills theory [9] predict different shear relaxation times $\tau_\pi = 3/2\pi T$ and $(2 - \ln 2)/2\pi T$, respectively, for $\eta/s = 1/4\pi$, the bulk relaxation time τ_Π remains completely ambiguous. Hence ad hoc choices have been made for the value of τ_Π in hydrodynamic studies [10–13].

Lattice QCD studies for gluonic plasma in fact predict large values of bulk viscosity to entropy density ratio, ζ/s, of about (6-25) $\eta/s|_{\rm KSS}$ near the QCD phase-transition temperature T_c [14]. This would translate into large values of the bulk pressure and bulk relaxation time, and may affect the evolution of the system significantly [11, 12]. Further, the large bulk pressure could result in a negative longitudinal pressure leading to mechanical instabilities (cavitation) whereby the fluid breaks up into droplets [13, 15, 16]. Thus the theoretical uncertainties arising from the absence of reliable estimates for the second-order transport coefficients should be eliminated for a proper understanding of the system evolution.

We present here a formal derivation of the dissipative hydrodynamic equations where all the second-order transport coefficients get determined uniquely within a single theoretical framework. This is achieved by invoking the second law of thermodynamics for the generalized entropy four-current expressed in terms of the phase-space distribution function given by Grad's 14-moment approximation. Significance of these coefficients is demonstrated in one-dimensional scaling expansion of the viscous medium.

Hydrodynamic evolution of a medium is governed by the conservation equations for the energy-momentum tensor and particle current [17]

$$T^{\mu\nu} = \int dp\ p^\mu p^\nu (f + \bar{f}) = \epsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu},$$
$$N^\mu = \int dp\ p^\mu (f - \bar{f}) = n u^\mu + n^\mu, \tag{1}$$

where $dp = gd\mathbf{p}/[(2\pi)^3\sqrt{\mathbf{p}^2+m^2}]$, g and m being the degeneracy factor and particle rest mass, p^μ is the particle four-momentum, $f \equiv f(x,p)$ is the phase-space distribution function for particles and $\bar{f}$ for antiparticles. The above integral expressions assume the system to be dilute so that the effects of interaction are small [17]. In the above tensor decompositions, ϵ, P, n are respectively energy density, pressure, net number density, and the dissipative quantities are the bulk viscous pressure (Π), shear stress tensor ($\pi^{\mu\nu}$) and particle diffusion current (n^μ). Here $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is the projection operator on the three-space orthogonal to the hydrodynamic four-velocity u^μ defined in the Landau frame: $T^{\mu\nu}u_\nu = \epsilon u^\mu$.

Energy-momentum conservation, $\partial_\mu T^{\mu\nu} = 0$ and current conservation, $\partial_\mu N^\mu = 0$ yield the fundamental evolution equations for ϵ, u^μ and n.

$$\begin{aligned} D\epsilon + (\epsilon+P+\Pi)\partial_\mu u^\mu - \pi^{\mu\nu}\nabla_{(\mu}u_{\nu)} &= 0, \\ (\epsilon+P+\Pi)Du^\alpha - \nabla^\alpha(P+\Pi) + \Delta^\alpha_\nu\partial_\mu\pi^{\mu\nu} &= 0, \\ Dn + n\partial_\mu u^\mu + \partial_\mu n^\mu &= 0. \end{aligned} \tag{2}$$

We use the standard notation $A^{(\alpha}B^{\beta)} = (A^\alpha B^\beta + A^\beta B^\alpha)/2$, $D = u^\mu\partial_\mu$, and $\nabla^\alpha = \Delta^{\mu\alpha}\partial_\mu$. Even if the equation of state is given, the system of Eqs. (2) is not closed unless the evolution equations for the dissipative quantities Π, $\pi^{\mu\nu}$, n^μ are specified.

Traditionally the dissipative equations have been obtained by invoking the second law of thermodynamics, viz., $\partial_\mu S^\mu \geq 0$, where the entropy four-current S^μ is given by [3, 7, 8]

$$\begin{aligned} S^\mu &= P\beta u^\mu - \alpha N^\mu + \beta u_\nu T^{\mu\nu} - Q^\mu(\delta N^\mu, \delta T^{\mu\nu}) \\ &= su^\mu - \frac{\mu n^\mu}{T} - \left(\beta_0\Pi^2 - \beta_1 n_\nu n^\nu + \beta_2\pi_{\rho\sigma}\pi^{\rho\sigma}\right)\frac{u^\mu}{2T} \\ &\quad - \left(\alpha_0\Pi\Delta^{\mu\nu} + \alpha_1\pi^{\mu\nu}\right)\frac{n_\nu}{T}. \end{aligned} \tag{3}$$

Here $\beta = 1/T$ is the inverse temperature, μ is the chemical potential, $\alpha = \beta\mu$, and Q^μ is a function of deviations from local equilibrium. The second equality is obtained by using the definition of the equilibrium entropy density $s = \beta(\epsilon + P - \mu n)$ and Taylor-expanding Q^μ to second order in dissipative fluxes. In this expansion, $\beta_i(\epsilon,n) \geq 0$ and $\alpha_i(\epsilon,n) \geq 0$ are the thermodynamic coefficients corresponding to pure and mixed terms. These coefficients can be obtained within the kinetic theory approach such as the IS theory [3]. However, it is important to note that they cannot be determined solely from thermodynamics using Eq. (3) and as a consequence the evolution equations remain incomplete.

In contrast to the above approach, our starting point for the derivation of the dissipative evolution equations is the entropy four-current expression generalized from Boltzmann's H-function:

$$\begin{aligned} S^\mu_{r=0} &= -\int dp\ p^\mu\left[f\left(\ln f - 1\right) + (f \to \bar{f})\right], \\ S^\mu_{r=\pm1} &= -\int dp\ p^\mu\left[\left(f\ln f + r\tilde{f}\ln\tilde{f}\right) + (f \to \bar{f})\right], \end{aligned} \tag{4}$$

where $\tilde{f} \equiv 1 - rf$ and $r = 1, -1, 0$ for Fermi, Bose, and Boltzmann gas, respectively. The divergence of $S^\mu_{r=0,\pm1}$ leads to

$$\partial_\mu S^\mu = -\int dp\ p^\mu\left[(\partial_\mu f)\ln(f/\tilde{f}) + (f \to \bar{f})\right]. \tag{5}$$

For small departures from equilibrium, f and $\bar{f}$ can be written as $f = f_0 + \delta f$ and $\bar{f} = \bar{f}_0 + \delta\bar{f}$. The equilibrium distribution functions are defined as $f_0 = [\exp(\beta u\cdot p - \alpha) + r]^{-1}$ and $\bar{f}_0 = [\exp(\beta u\cdot p + \alpha) + r]^{-1}$, where $\beta = 1/T$ and $\alpha = \mu/T$ are obtained from the equilibrium matching conditions $n \equiv n_0$ and $\epsilon \equiv \epsilon_0$.

To proceed further, we take recourse to Grad's 14-moment approximation [18] for the single particle distribution in orthogonal basis [19, 20]

$$f = f_0 + f_0\tilde{f}_0\phi, \quad \phi = \lambda_\Pi\Pi + \lambda_n n_\alpha p^\alpha + \lambda_\pi\pi_{\alpha\beta}p^\alpha p^\beta, \tag{6}$$

and similarly for $\bar{f}$. The coefficients $(\lambda_\Pi, \lambda_n, \lambda_\pi)$ are assumed to be independent of four-momentum p^μ and are functions of $(\epsilon, \alpha, \beta)$. From Eqs. (5) and (6), we get

$$\begin{aligned} \partial_\mu S^\mu = -\int dp\ p^\mu\Big[(\partial_\mu f)\left\{\ln\left(\frac{f_0}{\tilde{f}_0}\right) + \ln\left(1 + \frac{\phi}{1 - rf_0\phi}\right)\right\} \\ + (f \to \bar{f},\ f_0 \to \bar{f}_0)\Big]. \end{aligned} \tag{7}$$

The ϕ-independent terms on the right vanish due to energy-momentum and current conservation equations. To obtain second-order evolution equations for dissipative quantities, one should consider S^μ up to the same order. Hence $\partial_\mu S^\mu$ necessarily becomes third-order. Expanding the ϕ-dependent terms in Eq. (7) and retaining all terms up to third order in gradients (where ϕ is linear in dissipative quantities), we get

$$\begin{aligned} \partial_\mu S^\mu = -\int dp\ p^\mu\Big[\Big\{\phi\,(\partial_\mu f_0) - \phi^2(\tilde{f}_0 - 1/2)(\partial_\mu f_0) \\ + \phi^2\partial_\mu(f_0\tilde{f}_0) + \phi f_0\tilde{f}_0(\partial_\mu\phi)\Big\} + (f_0 \to \bar{f}_0)\Big]. \end{aligned} \tag{8}$$

The various integrals in the above equation can be decomposed into hydrodynamic tensor degrees of freedom via the definitions:

$$\begin{aligned} I_\pm^{\mu_1\mu_2\cdots\mu_n} \equiv \int dp\ p^{\mu_1}\cdots p^{\mu_n}(f_0 \pm \bar{f}_0) = I^\pm_{n0}u^{\mu_1}\cdots u^{\mu_n} \\ + I^\pm_{n1}(\Delta^{\mu_1\mu_2}u^{\mu_3}\cdots u^{\mu_n} + \text{perms}) + \cdots, \end{aligned} \tag{9}$$

where 'perms' denotes all non-trivial permutations of the Lorentz indices. We similarly define $J_\pm^{\mu_1\mu_2\cdots\mu_n}$ and

$K_{\pm}^{\mu_1\mu_2\cdots\mu_n}$ where the momentum integrals are weighted with $f_0\tilde{f}_0 \pm (f_0 \to \bar{f}_0)$ and $f_0\tilde{f}_0^2 \pm (f_0 \to \bar{f}_0)$, and are tensor decomposed with coefficients $J_{nq}^{\pm}$ and $K_{nq}^{\pm}$, respectively. All these coefficients can be obtained by suitable contractions of the integrals and are related to each other by

$$
\begin{aligned}
2K_{nq}^{\pm} &= J_{nq}^{\pm} + \frac{1}{\beta}\big[-J_{n-1,q-1}^{\pm} + (n-2q)J_{n-1,q}^{\pm}\big],\\
J_{nq}^{\pm} &= \frac{1}{\beta}\left[-I_{n-1,q-1}^{\pm} + (n-2q)I_{n-1,q}^{\pm}\right], \qquad (10)
\end{aligned}
$$

and also satisfy the differential relations

$$
\begin{aligned}
2K_{nq}^{\pm} &= J_{nq}^{\pm} - \frac{d}{d\beta}J_{n-1,q}^{\pm} = J_{nq}^{\pm} + \frac{d}{d\alpha}J_{nq}^{\pm},\\
J_{nq}^{\pm} &= -\frac{d}{d\beta}I_{n-1,q}^{\pm} = \frac{d}{d\alpha}I_{nq}^{\pm}. \qquad (11)
\end{aligned}
$$

With the help of these relations and Grad's 14-moment approximation, Eq. (8) reduces to

$$
\begin{aligned}
\partial_\mu S^\mu = &-\beta\Pi\Big[\theta + \beta_0\dot{\Pi} + \beta_{\Pi\Pi}\Pi\theta + \alpha_0\nabla_\mu n^\mu + \psi\alpha_{n\Pi}n_\mu\dot{u}^\mu\\
&+ \psi\alpha_{\Pi n}n_\mu\nabla^\mu\alpha\Big] - \beta n^\mu\Big[T\nabla_\mu\alpha - \beta_1\dot{n}_\mu - \beta_{nn}n_\mu\theta\\
&+ \alpha_0\nabla_\mu\Pi + \alpha_1\nabla_\nu\pi_\mu^\nu + \tilde{\psi}\alpha_{n\Pi}\Pi\dot{u}_\mu + \tilde{\psi}\alpha_{\Pi n}\Pi\nabla_\mu\alpha\\
&+ \tilde{\chi}\alpha_{\pi n}\pi_\mu^\nu\nabla_\nu\alpha + \tilde{\chi}\alpha_{n\pi}\pi_\mu^\nu\dot{u}_\nu\Big] + \beta\pi^{\mu\nu}\Big[\sigma_{\mu\nu} - \beta_2\dot{\pi}_{\mu\nu}\\
&- \beta_{\pi\pi}\theta\pi_{\mu\nu} - \alpha_1\nabla_{\langle\mu}n_{\nu\rangle} - \chi\alpha_{\pi n}n_{\langle\mu}\nabla_{\nu\rangle}\alpha\\
&- \chi\alpha_{n\pi}n_{\langle\mu}\dot{u}_{\nu\rangle}\Big], \qquad (12)
\end{aligned}
$$

where α_i, β_i, α_{XY}, β_{XX} are known functions of β, α and the integral coefficients $I_{nq}^{\pm}$, $J_{nq}^{\pm}$ and $K_{nq}^{\pm}$. Two new parameters ψ and χ with $\tilde{\psi} = 1-\psi$ and $\tilde{\chi} = 1-\chi$ are introduced to 'share' the contributions stemming from the cross terms of Π and $\pi^{\mu\nu}$ with n^μ.

The second law of thermodynamics, $\partial_\mu S^\mu \geq 0$, is guaranteed to be satisfied if we impose linear relationships between thermodynamical fluxes and extended thermodynamic forces, leading to the following evolution equations for bulk, charge current and shear

$$
\begin{aligned}
\Pi = &-\zeta\Big[\theta + \beta_0\dot{\Pi} + \beta_{\Pi\Pi}\Pi\theta + \alpha_0\nabla_\mu n^\mu\\
&+ \psi\alpha_{n\Pi}n_\mu\dot{u}^\mu + \psi\alpha_{\Pi n}n_\mu\nabla^\mu\alpha\Big], \qquad (13)\\
n^\mu = &\ \lambda\Big[T\nabla^\mu\alpha - \beta_1\dot{n}^{\langle\mu\rangle} - \beta_{nn}n^\mu\theta + \alpha_0\nabla^\mu\Pi\\
&+ \alpha_1\Delta_\rho^\mu\nabla_\nu\pi^{\rho\nu} + \tilde{\psi}\alpha_{n\Pi}\Pi\dot{u}^{\langle\mu\rangle} + \tilde{\psi}\alpha_{\Pi n}\Pi\nabla^\mu\alpha\\
&+ \tilde{\chi}\alpha_{\pi n}\pi_\nu^\mu\nabla^\nu\alpha + \tilde{\chi}\alpha_{n\pi}\pi_\nu^\mu\dot{u}^\nu\Big], \qquad (14)\\
\pi^{\mu\nu} = &\ 2\eta\Big[\sigma^{\mu\nu} - \beta_2\dot{\pi}^{\langle\mu\nu\rangle} - \beta_{\pi\pi}\theta\pi^{\mu\nu} - \alpha_1\nabla^{\langle\mu}n^{\nu\rangle}\\
&- \chi\alpha_{\pi n}n^{\langle\mu}\nabla^{\nu\rangle}\alpha - \chi\alpha_{n\pi}n^{\langle\mu}\dot{u}^{\nu\rangle}\Big], \qquad (15)
\end{aligned}
$$

with the coefficients of charge conductivity, bulk and shear viscosity, viz. $\lambda, \zeta, \eta \geq 0$. The notations, $A^{\langle\mu\rangle} = \Delta_\nu^\mu A^\nu$ and $B^{\langle\mu\nu\rangle} = \Delta_{\alpha\beta}^{\mu\nu}B^{\alpha\beta}$ represent space-like and traceless symmetric projections respectively, both orthogonal to u^μ, where $\Delta_{\alpha\beta}^{\mu\nu} = [\Delta_\alpha^\mu\Delta_\beta^\nu + \Delta_\beta^\mu\Delta_\alpha^\nu - (2/3)\Delta^{\mu\nu}\Delta_{\alpha\beta}]/2$. It may be noted that although the forms of the Eqs. (13)-(15) are the same as in the standard Israel-Stewart theory [3, 7], all the transport coefficients are explicitly determined in the present derivation:

$$
\begin{aligned}
&\beta_0 = \lambda_\Pi^2 J_{10}^{+}/\beta, \quad \beta_1 = -\lambda_n^2 J_{31}^{+}/\beta, \quad \beta_2 = 2\lambda_\pi^2 J_{52}^{+}/\beta,\\
&\alpha_0 = \lambda_\Pi\lambda_n J_{21}^{+}/\beta, \quad \alpha_1 = -2\lambda_\pi\lambda_n J_{42}^{+}/\beta. \qquad (16)
\end{aligned}
$$

As a consequence, the relaxation times defined as,

$$
\tau_\Pi = \zeta\,\beta_0, \quad \tau_n = \lambda\,\beta_1, \quad \tau_\pi = 2\,\eta\,\beta_2, \qquad (17)
$$

can be obtained directly. With $\lambda_\Pi = -1/J_{21}^{+}$, $\lambda_n = 1/J_{21}^{-}$, $\lambda_\pi = 1/(2J_{42}^{+})$, $n = I_{10}^{-}$, $\epsilon = I_{20}^{+}$, and $P = -I_{21}^{+}$, the expressions for $\beta_1, \alpha_0, \alpha_1$ simplify to

$$
\beta_1 = (\epsilon + P)/n^2, \quad \alpha_0 = \alpha_1 = 1/n. \qquad (18)
$$

For a classical Boltzmann gas ($\tilde{f}_0 = 1$), the coefficients β_0 and β_2 take the simple forms

$$
\beta_0 = 1/P, \quad \beta_2 = 3/(\epsilon + P) + m^2\beta^2 P/[2(\epsilon + P)^2]. \qquad (19)
$$

Equations (13)-(15) in conjunction with the second-order transport coefficients (18) and (19) constitute one of the main results in the present work. These coefficients are obtained consistently within the same theoretical framework. In contrast, in the standard derivation from entropy principles [3], the transport coefficients have to be estimated from an alternate theory. For instance, in the IS derivation based on kinetic theory, these involve complicated expressions which in the photon limit ($m\beta \to 0$) reduce to [21]

$$
\beta_0^{IS} = 216/(m^4\beta^4 P), \quad \beta_2^{IS} = 3/4P. \qquad (20)
$$

An alternate derivation from kinetic theory (KT) using directly the definition of dissipative currents yields [19]

$$
\begin{aligned}
\beta_0^{KT} = &\Big[\left(\frac{1}{3} - c_s^2\right)(\epsilon + P) - \frac{2}{9}(\epsilon - 3P)\\
&- \frac{m^4}{9}\left\langle (u.p)^{-2}\right\rangle\Big]^{-1},\\
\beta_2^{KT} = &\frac{1}{2}\left[\frac{4P}{5} + \frac{1}{15}(\epsilon - 3P) - \frac{m^4}{15}\left\langle (u.p)^{-2}\right\rangle\right]^{-1}, \qquad (21)
\end{aligned}
$$

where c_s is the speed of sound and $\langle\cdots\rangle \equiv \int dp(\cdots)f_0$. A field-theoretical (FT) approach gives [22]

$$
\begin{aligned}
\beta_0^{FT} &= \left[\left(\frac{1}{3} - c_s^2\right)(\epsilon + P) - \frac{a}{9}(\epsilon - 3P)\right]^{-1},\\
\beta_2^{FT} &= 1/[2(3-a)P], \qquad (22)
\end{aligned}
$$

where $a = 2$ for charged scalar bosons and $a = 3$ for fermions. We find that our expression for β_2 (Eq. (19))

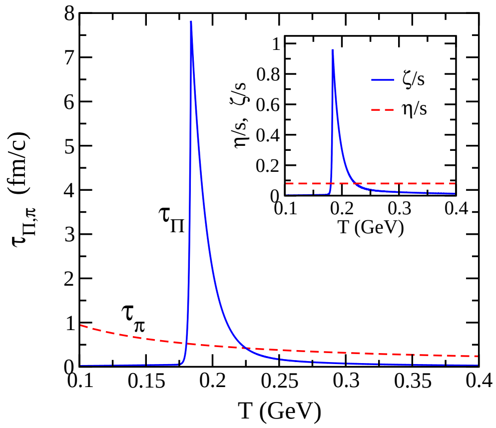

FIG. 1: (Color online) Temperature dependence of bulk and shear relaxation times. Inset shows ζ/s (see text) and $\eta/s = 1/4\pi$.

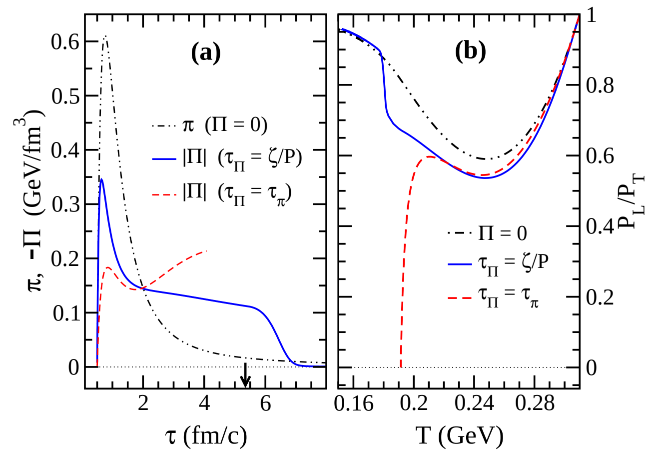

FIG. 2: (Color online) (a) Time evolution of shear stress in the absence of bulk ($\Pi = 0$) and magnitude of bulk stress for $\tau_\Pi = \zeta/P$ and $\tau_\Pi = \tau_\pi$. The arrow indicates the time when T_c is reached. (b) Temperature dependence of pressure anisotropy, P_L/P_T, for these three cases. The results are for initial $T = 310$ MeV, $\tau_0 = 0.5$ fm/c and $\eta/s = 1/4\pi$. The evolution is stopped when P_L vanishes.

in the massless limit, agrees with the IS result (Eq. (20)) and also with those obtained in Refs. [8, 23]. Thus the shear relaxation times τ_π (Eq. (17)) obtained here and in these studies are also identical. As β_0 in Eqs. (20)-(22) diverge in the massless limit, so does the bulk relaxation time τ_Π (Eq. (17)), thereby stopping the evolution of the bulk pressure. It is important to note that β_0 in Eq. (19) and hence τ_Π in the present calculation remain finite in this limit. A detailed comparison of IS, KT and FT results can be found in [24]. The two parameters ψ and χ occurring in Eq. (12) remain undetermined as in [3]; however, these do not contribute to the scaling expansion.

To demonstrate the numerical significance of the new coefficients derived here, we consider the evolution equations in the boost-invariant Bjorken hydrodynamics at vanishing net baryon number density [25]. In terms of the coordinates (τ, x, y, η) where $\tau = \sqrt{t^2 - z^2}$ and $\eta = \tanh^{-1}(z/t)$, the initial four-velocity becomes $u^\mu = (1, 0, 0, 0)$. For this scenario $n^\mu = 0$ and the evolution equations for ϵ, $\pi \equiv -\tau^2\pi^{\eta\eta}$ and Π reduce to

$$\frac{d\epsilon}{d\tau} = -\frac{1}{\tau}\left(\epsilon + P + \Pi - \pi\right), \tag{23}$$

$$\tau_\pi \frac{d\pi}{d\tau} = \frac{4\eta}{3\tau} - \pi - \frac{4\tau_\pi}{3\tau}\pi, \tag{24}$$

$$\tau_\Pi \frac{d\Pi}{d\tau} = -\frac{\zeta}{\tau} - \Pi - \frac{4\tau_\Pi}{3\tau}\Pi. \tag{25}$$

Noting that $\beta_0 = 1/P$, $\beta_2 = 3/(\epsilon + P)$ and $s = (\epsilon + P)/T$, the relaxation times defined in Eq. (17) reduce to

$$\tau_\Pi = \frac{\epsilon + P}{PT}\left(\frac{\zeta}{s}\right), \quad \tau_\pi = \frac{6}{T}\left(\frac{\eta}{s}\right). \tag{26}$$

We have used the state-of-the-art equation of state [26], which is based on a recent lattice QCD result [27]. For ζ/s at $T \geq T_c \approx 184$ MeV, we use the parametrized form [13] of the lattice QCD results of Meyer [14] which suggest a peak near T_c. At $T < T_c$, the sharp drop in ζ/s reflects its extremely small value found in the hadron resonance gas model [28]; see inset of Fig. 1. For the η/s ratio, we use the minimal KSS bound [6] value of $1/4\pi$.

In the absence of any reliable prediction for the bulk relaxation time τ_Π, it has been customary to keep it fixed or set it equal to the shear relaxation time τ_π or parametrize it in such a way that it captures critical slowing-down of the medium near T_c due to growing correlation lengths [10–13]. Since ζ/s has a peak near the phase transition, the τ_Π obtained here (Eq. (26)) and shown in Fig. 1, *naturally* captures the phenomenon of critical slowing-down.

The evolution equations (23)-(25) are solved simultaneously with an initial temperature $T_0 = 310$ MeV [13] and initial time $\tau_0 = 0.5$ fm/c typical for the RHIC energy scan. We take initial values for bulk stress and shear stress, $\Pi = \pi = 0$ GeV/fm^3 which corresponds to an isotropic initial pressure configuration.

Figure 2(a) shows time evolution of the shear pressure π and the magnitude of the bulk pressure Π. At early times $\tau \lesssim 2$ fm/c or equivalently at $T \gtrsim 1.2T_c$, shear dominates bulk. This implies that eccentricity-driven elliptic flow which develops early in the system would be controlled more by the shear pressure [12]. At later times (when $T \sim T_c$), the large value of ζ/s makes the bulk pressure dominant. This leads to sizeable entropy generation (Eq. (12)) and consequently enhanced particle production.

Figure 2(a) also compares the Π evolution for bulk relaxation time, τ_Π, calculated from Eq. (26) (solid line) and $\tau_\Pi = \tau_\pi$ (dashed line). At early times, the larger value of τ_Π in the latter case (see Fig. 1) results in a relatively smaller growth of $|\Pi|$ as evident from Eq. (25).

Near T_c, the rapid increase in ζ/s causes $|\Pi|$ to increase. Subsequently the longitudinal pressure $P_L = (P+\Pi-\pi)$ vanishes leading to cavitation [10, 13, 15, 16]. In contrast, with our τ_Π, this rise in ζ/s is overcompensated by a faster increase in τ_Π thereby slowing down the evolution of Π. This behavior prevents the onset of cavitation and guarantees the applicability of hydrodynamics with bulk and shear up to temperatures well below T_c into the hadronic phase. Furthermore, this slowing down of the medium followed by its rapid expansion, has the right trend to explain the identical-pion correlation measurements (Hanbury Brown-Twiss puzzle) [29, 30].

The absence of cavitation in our calculation is clearly evident in Fig. 2(b) which shows the variation of pressure anisotropy, $P_L/P_T = (P+\Pi-\pi)/(P+\Pi+\pi/2)$, with temperature. Near T_c, the longitudinal pressure P_L vanishes if one assumes $\tau_\Pi = \tau_\pi$ (dashed line) leading to cavitation, whereas it is found to be positive for all temperatures with τ_Π derived here (solid line). In fact, we have found that in the latter case, cavitation is completely avoided for the entire range of ζ/s values ($0.5 < \zeta/s < 2.0$ near T_c) estimated in lattice QCD [14]. The sizeable difference between the $\Pi = 0$ case (dot-dashed line) and the $\tau_\Pi = \zeta/P$ case (solid line) clearly underscores the importance of bulk pressure near T_c, which can have significant implications for the elliptic flow v_2 [11] thus affecting the extraction of η/s. Further, the large bulk pressure when incorporated in the freezeout prescription could also affect the final particle abundances and spectra.

We have also found that the evolution of Π is insensitive to the choice of initial conditions such as $\Pi(\tau_0) = 0$ and the Navier-Stokes value $-\zeta(T_0)/\tau_0$. This is due to very small τ_Π at early times (or higher temperatures) which causes Π to quickly lose the memory of its initial condition and to relax to the same value at $\tau \gtrsim 1$ fm/c.

To summarize, we have presented a new derivation of the relativistic dissipative hydrodynamic equations from entropy considerations. We arrive at the same form of dissipative evolution equations as in the standard derivation but with all second-order transport coefficients such as the relaxation times and the entropy flux coefficients determined consistently within the same framework. We find that in the Bjorken scenario, although the bulk pressure can be large, the relaxation time derived here prevents the onset of cavitation due to the critical slowing down of bulk evolution near T_c.

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